

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Pearson Edexcel International GCSE		Centre Number 	Candidate Number
Thursday 6 June 2019			
Morning (Time: 2 hours 30 minutes)		Paper Reference 4MB1/02R	
Mathematics B Paper 2R			
You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.			Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- **Calculators may be used.**

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.

Turn over ►

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Answer ALL ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 $\mathcal{E}$ is the universal set and A , B and C are three sets where

$\mathcal{E} = \{\text{positive integers less than } 13\}$

$A = \{\text{multiples of } 5\}$

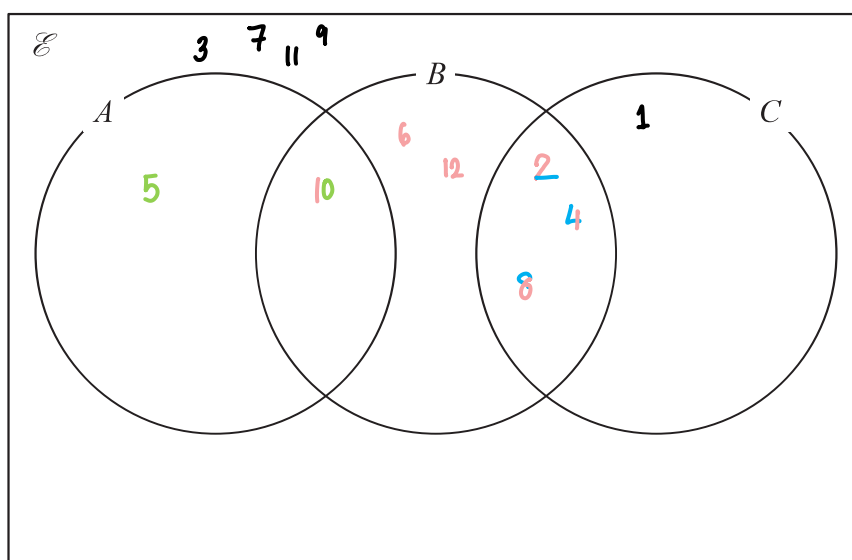
$B = \{\text{even numbers}\}$

$C = \{\text{factors of } 8\}$

The Venn diagram below can be used to show these sets.

- (a) Complete the Venn diagram for the sets $\mathcal{E}$, A , B and C .

(3)



List the elements of the sets

- (b) $B \cap C$

(1)

- (c) $A \cup B$

(1)

- (d) $(A \cup B \cup C)'$

(1)

Find

- (e) $n(B \cup C)$

(1)

- (f) $n(B' \cap C')$

(1)

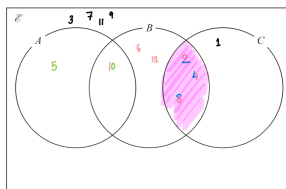


Question 1 continued

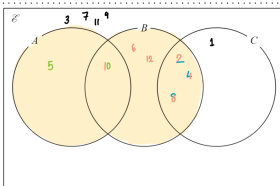
(a) Completed Venn diagram.

(b) $B \cap C$ is the intersection of B and C.

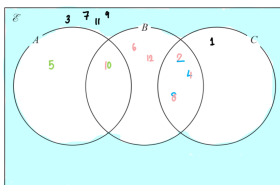
$$B \cap C: \{2, 4, 8\}$$

(c) $A \cup B$ is all the values in A or B.

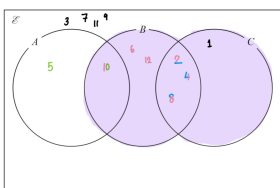
$$A \cup B: \{2, 4, 5, 6, 8, 10, 12\}$$

(d) $(A \cup B \cup C)'$ is all the values not in A or B or C.

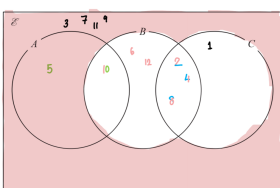
$$(A \cup B \cup C)': \{3, 7, 9, 11\}$$

(e) $n(B \cup C)$ is the number of values in B or C. These values are $\{1, 2, 4, 6, 8, 10, 12\}$ which are 7 numbers.

$$n(B \cup C): 7$$

(f) $n(B' \cap C')$ is the number of values not in B and not in C. These are $\{3, 5, 7, 9, 11\}$ which are 5 values

$$n(B' \cap C'): 5$$



(Total for Question 1 is 8 marks)



- 2 (a) (i) Solve the inequality $6x + 10 \geq 2x + 2$ (2)
- (ii) Represent your solution on the number line on the next page. (1)
- (b) (i) Solve the inequality $(4x + 3)(2x - 1) < (6x + 5)(x - 2)$ (5)
- (ii) Represent your solution on the same number line. (1)
- (c) Write down the set of values for which
both $6x + 10 \geq 2x + 2$ **and** $(4x + 3)(2x - 1) < (6x + 5)(x - 2)$ (1)

(a) (i)

$$\begin{array}{l}
 6x + 10 \geq 2x + 2 \\
 \xrightarrow{-2x} 4x + 10 \geq 2 \\
 \xrightarrow{-10} 4x \geq -8 \\
 \xrightarrow{\div 2} x \geq -2
 \end{array}$$

(ii) Since x is greater than or equal to -2 , there needs to be a full circle with the arrow pointing to the right.

(b) (i)

$$\begin{array}{l}
 (4x + 3)(2x - 1) < (6x + 5)(x - 2) \\
 \xrightarrow{\text{Expand}} 8x^2 - 4x + 6x - 3 < 6x^2 - 12x + 5x - 10 \\
 \xrightarrow{\text{Simplify}} 8x^2 + 2x - 3 < 6x^2 - 7x - 10 \\
 \xrightarrow{-6x^2} 2x^2 + 2x - 3 < -7x - 10 \\
 \xrightarrow{+7x} 2x^2 + 9x - 3 < -10 \\
 \xrightarrow{+10} 2x^2 + 9x + 7 < 0 \\
 \xrightarrow{\text{Factorise}} \frac{2x^2 + 9x + 7}{2} < 0 \\
 \quad \quad \quad 2x^2 = 14 \\
 \quad \quad \quad (2x + 7)(x + 1) < 0
 \end{array}$$

$$(2x + 7)(x + 1) < 0$$

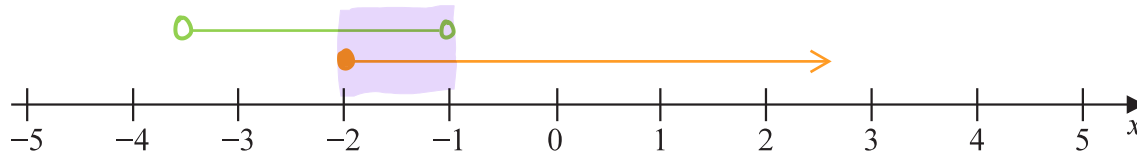
$$\begin{array}{l}
 2x + 7 = 0 \\
 \xrightarrow{-7} 2x = -7 \\
 \xrightarrow{\div 2} x = -7/2
 \end{array}$$

$$\begin{array}{l}
 x + 1 = 0 \\
 \xrightarrow{-1} x = -1
 \end{array}$$

→ You need to find the roots to sketch a diagram.

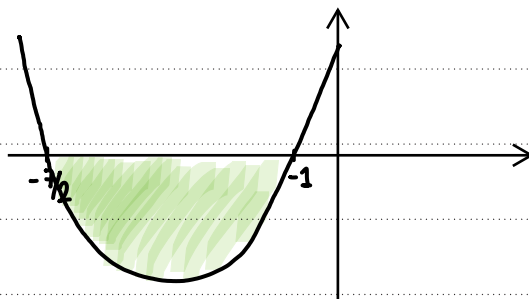


Question 2 continued



(b)(i) Continued...

You now sketch a diagram



Since $2x^2 + 9x + 7$ is less than 0, we have to consider all values taken by the graph which are below the x -axis.

$$\therefore -\frac{7}{2} < x < -1$$

(ii) Since the $<$ sign does not have an equal, we can use an open circle.

(c) $-2 \leq x < -1$

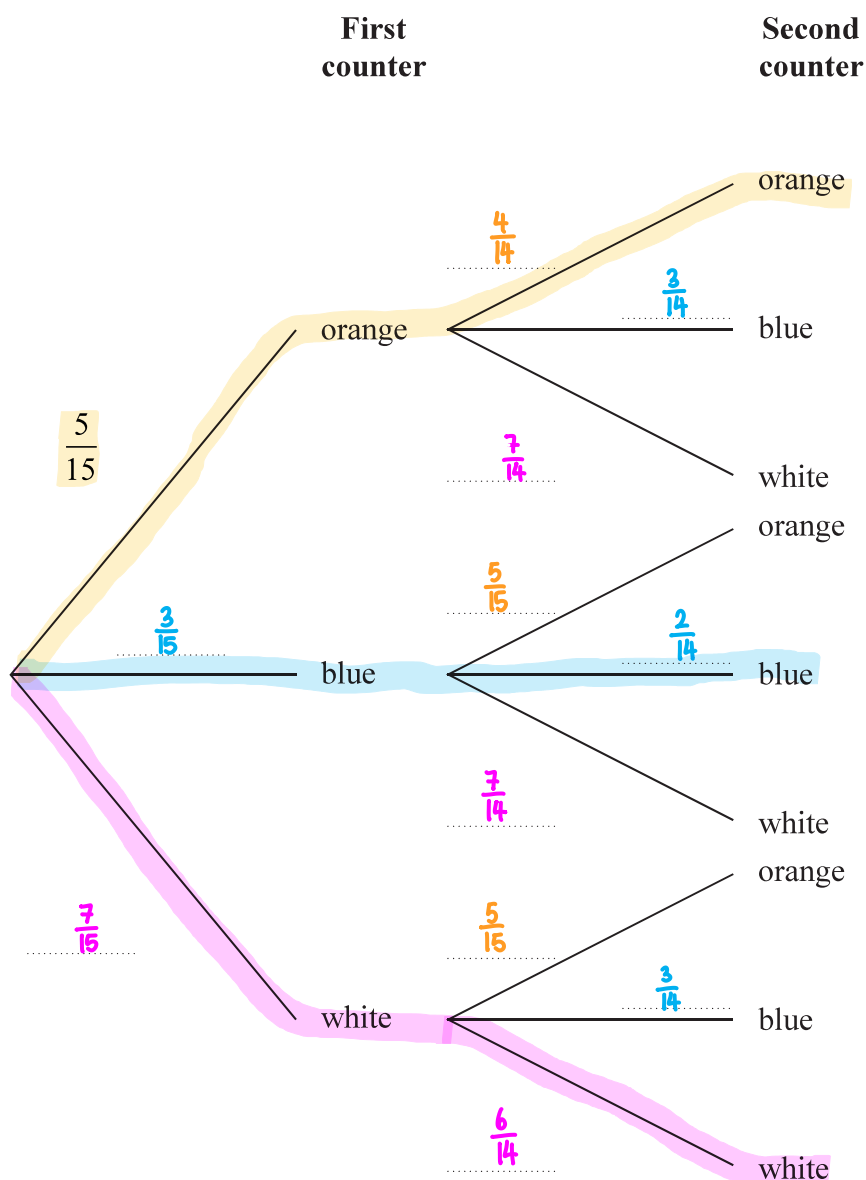
(Total for Question 2 is 10 marks)



- 3 A bag contains 15 counters.
There are
5 orange counters
3 blue counters
7 white counters.

Priya takes at random 2 counters from the bag.

- (a) Complete the probability tree diagram.



(3)

- (b) Calculate the probability that Priya takes 2 white counters.

(2)

- (c) Calculate the probability that the counters Priya takes are not of the same colour.

(3)



Question 3 continued

$$(b) P(2 \text{ white}) = P(\text{white}) \times P(\text{white})$$

$$= \frac{7}{15} \times \frac{6}{14}$$

$$= \frac{42}{210}$$

$$= \frac{1}{5}$$

(c) The easiest way to answer this question is to calculate the probability of picking all the same colours and subtracting from 1.

$$P(2 \text{ oranges}) = P(\text{orange}) \times P(\text{orange})$$

$$= \frac{5}{15} \times \frac{4}{14}$$

$$= \frac{20}{210}$$

$$P(2 \text{ blue}) = P(\text{blue}) \times P(\text{blue})$$

$$= \frac{3}{15} \times \frac{2}{14}$$

$$= \frac{6}{210}$$

$$P(2 \text{ white}) = \frac{42}{210} \text{ (from part (b))}$$

$$P(\text{only same colour}) = P(2 \text{ oranges}) + P(2 \text{ blue}) + P(2 \text{ white})$$

$$= \frac{20}{210} + \frac{6}{210} + \frac{42}{210}$$

$$= \frac{68}{210}$$

$$= \frac{34}{105}$$

$$P(\text{not the same colour}) = 1 - \frac{34}{105}$$

$$= \frac{71}{105}$$

(Total for Question 3 is 8 marks)



- 4 The total amount of money that was spent last March by Marco's family was \$4200

The table below gives information about the amount Marco's family spent on each of four items last March.

Item	Amount
healthcare	\$336
insurance	\$504
food	\$546
travel	\$630
housing	
entertainment	
other items	

- (a) Calculate the percentage of the total amount that was spent on healthcare. (2)

The amount spent on travel was divided between Marco's car, his wife's car and other transport in the ratios 4 : 3 : 2

- (b) Calculate the amount, in \$, that was spent on Marco's car. (2)

The amount that was spent on food last March was 12% more than was spent on food last February.

- (c) Calculate the amount, in \$, that was spent on food last February. (3)

The amount of money spent on entertainment last March was 12.5% of the total amount of money that was not spent on healthcare, insurance, food and travel.

- (d) Calculate the amount, in \$, that was spent on entertainment. (3)

Marco draws a pie chart for the amounts his family spent on all the items last March.

- (e) Calculate the size of the angle, in degrees, for insurance. (2)

The angle in the pie chart for housing is 114°

- (f) Calculate the amount, in \$, that was spent on housing. (2)



Question 4 continued

(a) Here you need to divide the amount spent on healthcare by the total spend and multiply by 100.

$$\frac{336}{4200} \times 100 = 8\%$$

(b) To find the amount spent on Marco's car, you first need to find the amount that one part stands for.

$$\frac{630}{4+3+2} = 70$$

One part stands for \$70 so the multiplier is 70.

Marco's car : his wife's car : other transport

$$4 : 3 : 2$$

$$280 : 210 : 140$$

Marco's car: \$280

(c) If we use the multiplier we can find the amount spent on food in February by working backwards.

Since the amount for food in March is 12% more than February, you count the amount in February as 100% and the 12% extra.

$$100\% + 12\% = 112\% = 1.12$$

$$\text{Amount spend on food in February} \times 1.12 = \text{Amount spent on food in March}$$

$$\text{Amount spend on food in February} \times 1.12 = 546$$

$$\text{Amount spend on food in February} = \frac{546}{1.12}$$

$$\text{Amount spend on food in February} = \$487.50$$

(d) You first need to find the total money spent on healthcare, insurance, food and travel.

$$336 + 504 + 546 + 630 = 2016$$

$$2016 \times \frac{12.5}{100} = \$252$$



Question 4 continued

(e) You can consider the total spend and the total angle of a circle to find the degrees taken by insurance.

$$\begin{array}{c} \$4200 : \$504 \\ \times \frac{360}{4200} \end{array} \rightarrow 360^\circ : 43.2^\circ \leftarrow \begin{array}{c} \times \frac{360}{4200} \end{array}$$

Ans: 43.2°

(f) Here, we have to use the same principle as in part (e).

$$\begin{array}{c} \text{Total : Housing} \\ 360^\circ : 114^\circ \\ \times \frac{4200}{360} \end{array} \rightarrow \$4200 : \$1330 \leftarrow \begin{array}{c} \times \frac{4200}{360} \end{array}$$

Ans: \$1330



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Question 4 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 4 is 14 marks)



5 Triangles A and B are drawn on the grid opposite.

(a) Describe fully the single transformation that maps triangle A onto triangle B .

(3)

Triangle A is reflected in the y -axis to give triangle C .

(b) On the grid, draw and label triangle C .

(1)

Triangle D is the image of triangle C under the transformation with matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

(c) On the grid, draw and label triangle D .

(3)

Triangle D is the image of triangle A under the transformation with matrix $\mathbf{N}$.

(d) Find the matrix $\mathbf{N}$.

(3)

(a) Enlargement of scale factor 2 with centre of enlargement $(-1, 5)$.

(b) Reflect A .

(c) Rule for multiplying a 1×2 by a 2×2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0(-2) + 2(4) \\ -2(-2) + 0(4) \end{pmatrix}$$

$$= \begin{pmatrix} 0+8 \\ 4+0 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0(-2) + 2(1) \\ -2(-2) + 0(1) \end{pmatrix}$$

$$= \begin{pmatrix} 0+2 \\ 4+0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

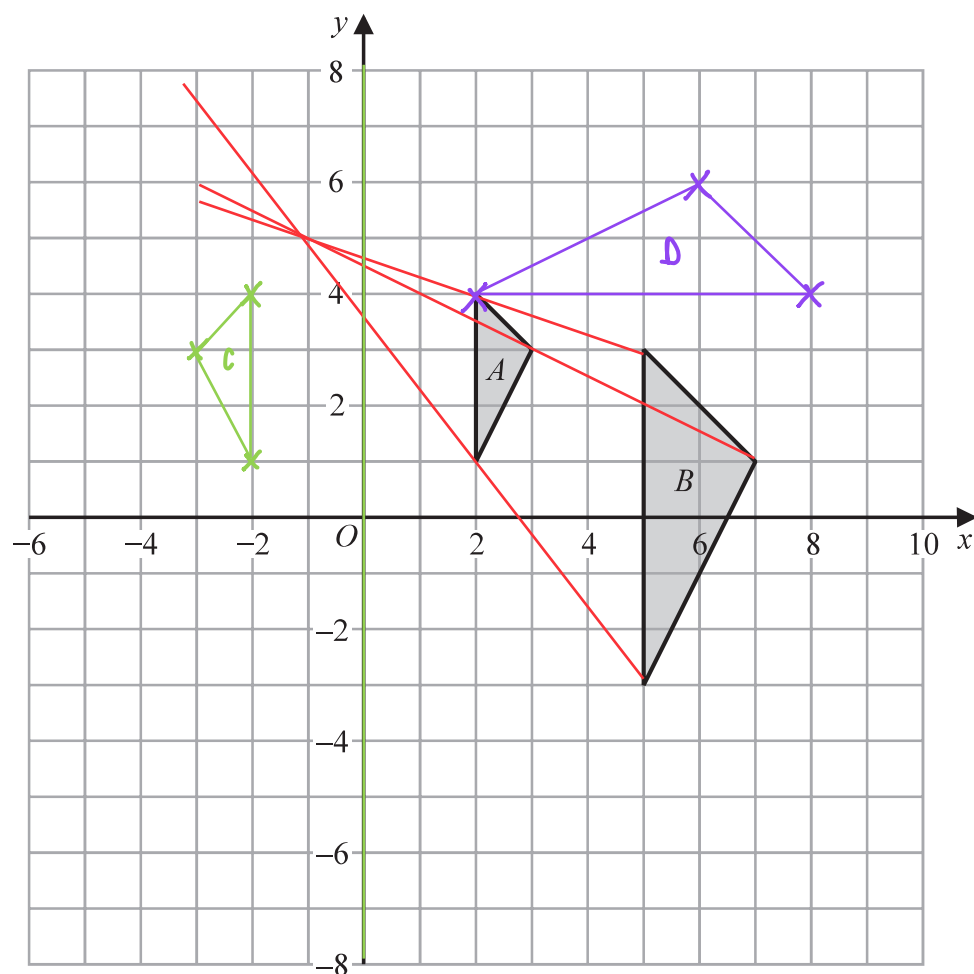
$$\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 0(-3) + 2(3) \\ -2(-3) + 0(3) \end{pmatrix}$$

$$= \begin{pmatrix} 0+6 \\ 6+0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$



Question 5 continued



(d) You now need to work backwards to find the matrix N .

$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} a(2) & b(4) \\ c(2) & d(4) \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$2a + 4b = 8$$

$$2c + 4d = 4$$

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} a(3) + b(3) \\ c(3) + d(3) \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

$$3a + 3b = 6$$

$$3c + 3d = 6$$

Now that you have two equations with two unknowns, you can find the unknowns using simultaneous equations.

Turn over for a spare grid if you need to redraw your triangles.



Question 5 continued

$$2a + 4b = 8$$

$$3a + 3b = 6$$

Method 1: Elimination

① Make the value in front of a or b equal. If making a's value equal:

$$\begin{array}{l} \times 3 \left(\begin{array}{l} 2a + 4b = 8 \\ 6a + 12b = 24 \end{array} \right) \times 3 \end{array}$$

$$\begin{array}{l} \times 2 \left(\begin{array}{l} 3a + 3b = 6 \\ 6a + 6b = 12 \end{array} \right) \times 2 \end{array}$$

② Since there are the same signs in front of a, you can subtract.

$$6a + 12b = 24$$

$$\underline{-6a + 6b = 12}$$

$$\begin{array}{l} 6b = 12 \\ \div 6 \rightarrow b = 2 \end{array}$$

③ Substitute your value of b into one of the original equations to find a value for a.

$$2a + 4(2) = 8$$

$$\begin{array}{l} -8 \left(\begin{array}{l} 2a + 8 = 8 \\ 2a = 0 \end{array} \right) -8 \\ \div 2 \rightarrow a = 0 \end{array}$$

$$2c + 4d = 4$$

$$3c + 3d = 6$$

Method 1: Elimination

① Make the value in front of c or d equal. If making c equal:

$$\begin{array}{l} \times 3 \left(\begin{array}{l} 2c + 4d = 4 \\ 6c + 12d = 12 \end{array} \right) \times 3 \end{array}$$

$$\begin{array}{l} \times 2 \left(\begin{array}{l} 3c + 3d = 6 \\ 6c + 6d = 12 \end{array} \right) \times 2 \end{array}$$

② Since the sign in front of c is the same, subtract.

$$6c + 12d = 12$$

$$\underline{-6c + 6d = 12}$$

$$\begin{array}{l} 6d = 0 \\ \div 6 \rightarrow d = 0 \end{array}$$

③ Substitute your value of d into one of the original equations for c.

$$2c + 4(0) = 4$$

$$\begin{array}{l} \div 2 \left(\begin{array}{l} 2c = 4 \\ c = 2 \end{array} \right) \div 2 \end{array}$$



Question 5 continued

Method 1: Substitution

① Make a or b the subject of one of your equations.

$$\begin{aligned} 2a + 4b &= 8 \\ -4b & \quad -4b \\ \hline 2a &= 8 - 4b \\ \div 2 & \quad \div 2 \\ \hline a &= 4 - 2b \end{aligned}$$

② Substitute into your other equation

$$\begin{aligned} 3(4 - 2b) + 3b &= 6 \\ \text{Simplify} & \quad 12 - 6b + 3b = 6 \\ & \quad 12 - 3b = 6 \\ -12 & \quad -12 \\ \hline -3b &= -6 \\ \div -3 & \quad \div -3 \\ \hline b &= 2 \end{aligned}$$

③ Substitute your value of b into one of the original equations for a value of a .

$$\begin{aligned} 2a + 4(2) &= 8 \\ 2a + 8 &= 8 \\ -8 & \quad -8 \\ \hline 2a &= 0 \\ \div 2 & \quad \div 2 \\ \hline a &= 0 \end{aligned}$$

Matrix $N = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

Method 2: Substitution

① Make c the subject of one of your equations.

$$\begin{aligned} 2c + 4d &= 4 \\ -4d & \quad -4d \\ \hline 2c &= 4 - 4d \\ \div 2 & \quad \div 2 \\ \hline c &= 2 - 2d \end{aligned}$$

② Substitute into your other equation

$$\begin{aligned} 3(2 - 2d) + 3d &= 6 \\ \text{Simplify} & \quad 6 - 6d + 3d = 6 \\ & \quad 6 - 3d = 6 \\ -6 & \quad -6 \\ \hline -3d &= 0 \\ \div -3 & \quad \div -3 \\ \hline d &= 0 \end{aligned}$$

③ Substitute your value of d into one of the original equations for a value of c .

$$\begin{aligned} 2c + 4(0) &= 4 \\ 2c &= 4 \\ \div 2 & \quad \div 2 \\ \hline c &= 2 \end{aligned}$$

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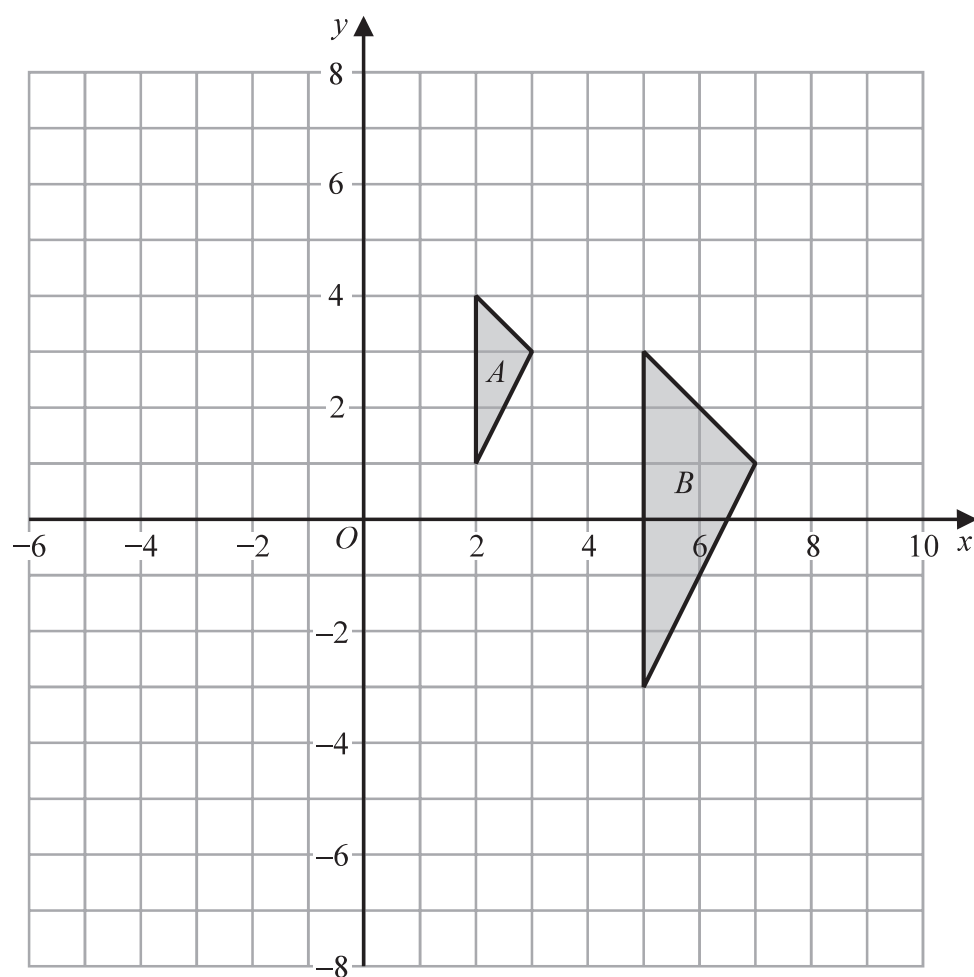
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Question 5 continued

Only use this grid if you need to redraw your triangles.



(Total for Question 5 is 10 marks)



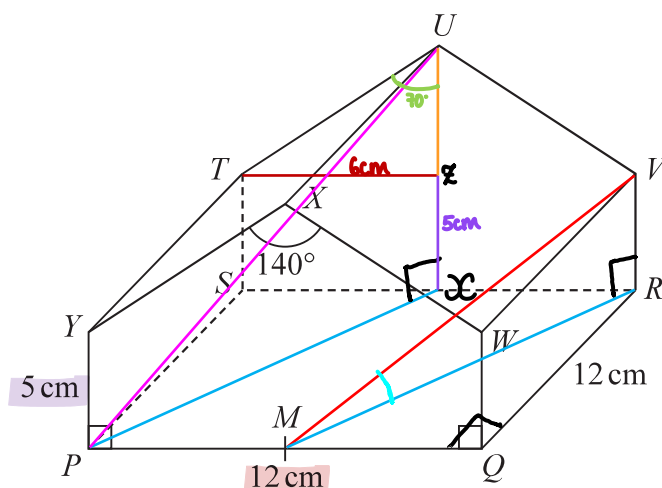


Diagram NOT
accurately drawn

Figure 1

Figure 1 shows a right pentagonal prism, with a horizontal square base $PQRS$ of side 12 cm.

$$YP = WQ = 5 \text{ cm}$$

$$\angle WXY = 140^\circ$$

$$\angle YPQ = \angle WQP = 90^\circ$$

M is the midpoint of PQ so that XM is an axis of symmetry of the pentagon.

(a) Calculate the size, in degrees to one decimal place, of $\angle VMR$.

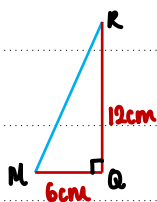
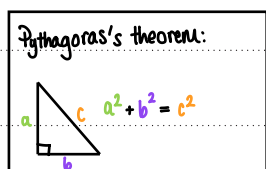
(3)

(b) Calculate the length, in cm to one decimal place, of PU .

(4)

(a) To find $\angle VMR$, you need to construct two right angled triangles and then use Pythagoras and trigonometry.

To use trigonometry on triangle VMR , you need the size of two sides. We know the size of VR is 5 cm and can find the RM using Pythagoras on ΔRQM .



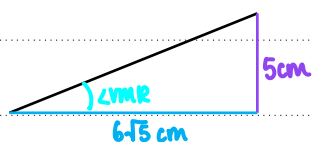
$$RQ^2 + QM^2 = RM^2$$

$$12^2 + 6^2 = RM^2$$

$$\sqrt{12^2 + 6^2} = RM$$

$$6.5 = RM$$

Now find $\angle VMR$ using trigonometry (S^oH C^oH T^oA)



Since you have the opposite and the adjacent, you can use $\tan$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



Question 6 continued

$$\tan \angle VNR = \frac{5}{6\sqrt{5}}$$

$$\angle VNR = \tan^{-1}\left(\frac{5}{6\sqrt{5}}\right)$$

$$\angle VNR = 20.43\dots$$

$$\approx 20.4^\circ$$

(b) To calculate the length of PU , you need to form a new triangle PXC and using the lengths PX and XC , use Pythagoras to find PU .

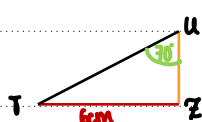
The length of PX is equal to the length of RM .

$$PX = 6\sqrt{5} \text{ cm}$$

The length XC is a combination of XZ and UZ .

$$XZ = 5 \text{ cm}$$

You can find ZU using trigonometry with $\triangle TUV$



$$\text{Since } \angle YXW = \angle TUV$$

$$\angle TUV = 140^\circ$$

And, since $\angle TUZ$ is half of $\angle TUV$,

$$\angle TUZ = 70^\circ$$

In $\triangle TUZ$, we know the angle, the opposite and we want to find the adjacent so we can use $\tan$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 70^\circ = \frac{6}{UZ}$$

$$UZ = \frac{6}{\tan 70^\circ}$$

$$UZ = 2.1838\dots$$



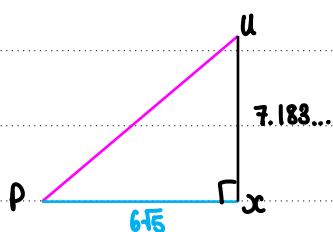
Question 6 continued

$$ux = x^2 + u^2$$

$$ux = 5 + 2.183...$$

$$ux = 7.1838... \text{ cm}$$

You can now use Pythagoras to find pu .



$$pu^2 = px^2 + xu^2$$

$$pu^2 = (6\sqrt{5})^2 + (7.1838...)^2$$

$$pu = \sqrt{(6\sqrt{5})^2 + (7.1838...)^2}$$

$$pu = 15.2186...$$

$$\approx 15.2 \text{ cm}$$



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Question 6 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 6 is 7 marks)



7 The equation of a curve is $y = 5 - \frac{x}{2} - x^2$

(a) Complete the table of values for $y = 5 - \frac{x}{2} - x^2$

x	-4	-3	-2	-1	0	1	2	3
y	-9	-2.5	2	4.5	0	3.5	0	-5.5

(2)

(b) On the grid opposite, plot the points from your completed table and join them to form a smooth curve.

(2)

(c) Using your curve, find an estimate of the maximum value, to one decimal place, of $y = 5 - \frac{x}{2} - x^2$

(1)

(d) Use your curve to find the range of values of x , to one decimal place, for which $5 - \frac{x}{2} - x^2 \geq 3$

(2)

(e) By drawing a suitable straight line on the grid, find estimates, to one decimal place, for the solutions of the equation $3 - x - x^2 = 0$

(3)

(a) Substitute the given x coordinates into the curve's equation for the y -coordinates.

$$x = -3$$

$$y = 5 - \frac{(-3)}{2} - (-3)^2$$

$$= -2.5$$

$$x = 0$$

$$y = 5 - \frac{(0)}{2} - (0)$$

$$= 5$$

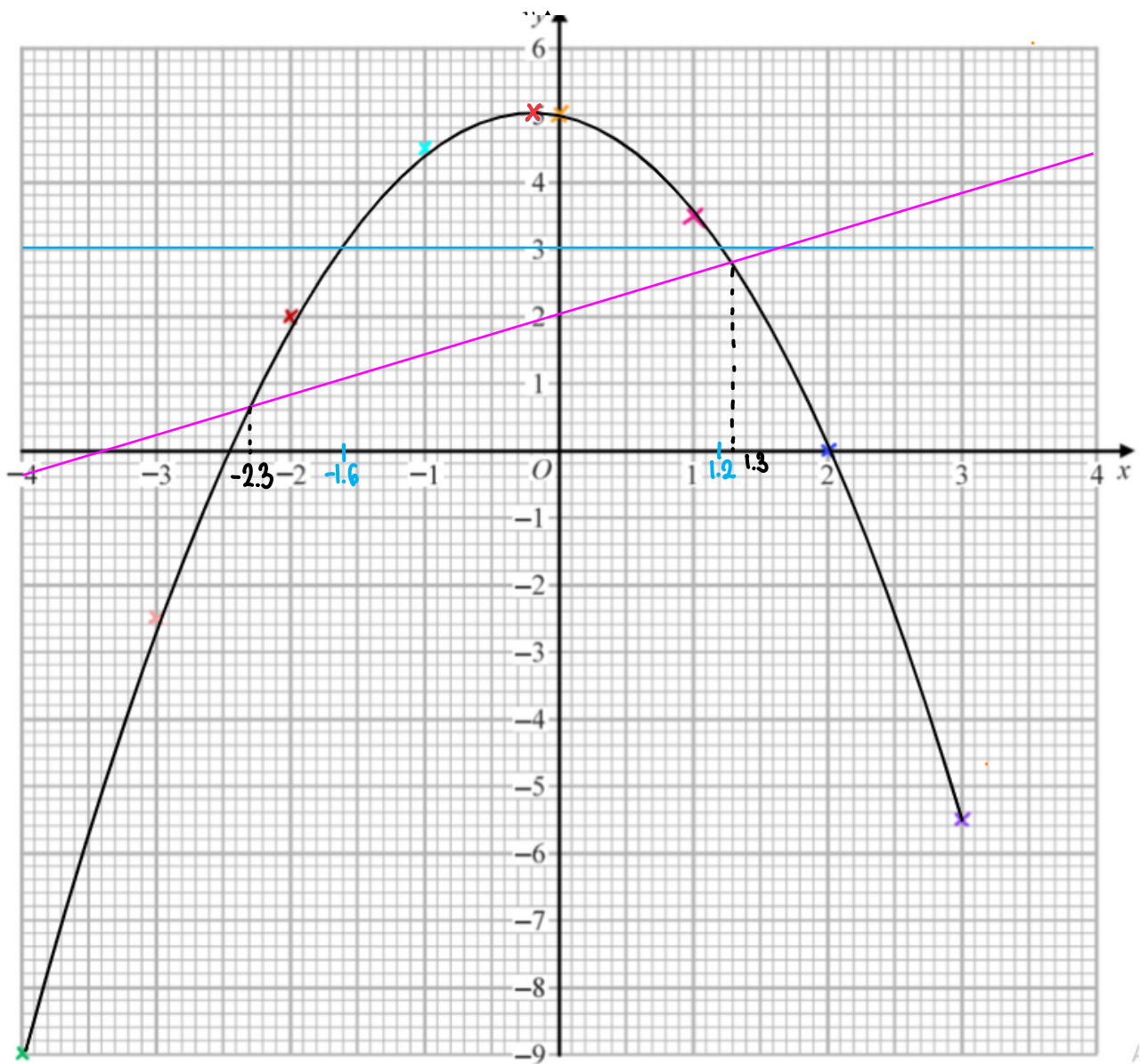
$$x = 3$$

$$y = 5 - \frac{(3)}{2} - (3)$$

$$= -5.5$$



Question 7 continued



(c) $x = 5.1$

(d) $5 - \frac{x}{2} - x^2 \geq 3$ means all values of y that are greater than or equal to 3.

$$-1.7 \leq x \leq 1.2$$

Turn over for a spare grid if you need to redraw your curve.



Question 7 continued

(e) You now have to find a suitable equation that if you subtract from $y = 5 - \frac{x}{2} - x^2$, you will get

$$y = 3 - x - x^2$$

$$5 - 0.5x - \cancel{x^2} - (mx + c) = 3 - x - \cancel{x^2}$$

$$5 - c = 3$$

$$-c = -2$$

$$c = 2$$

$$-0.5x - mx = -x$$

$$-0.5 - m = -1$$

$$-m = -0.5$$

$$m = 0.5$$

$$\therefore y = 0.5x + 2$$

You can now find the points of intersection of $y = 0.5x + 2$ and $y = 5 - \frac{x}{2} - x^2$

Ans: -2.3 and 1.3



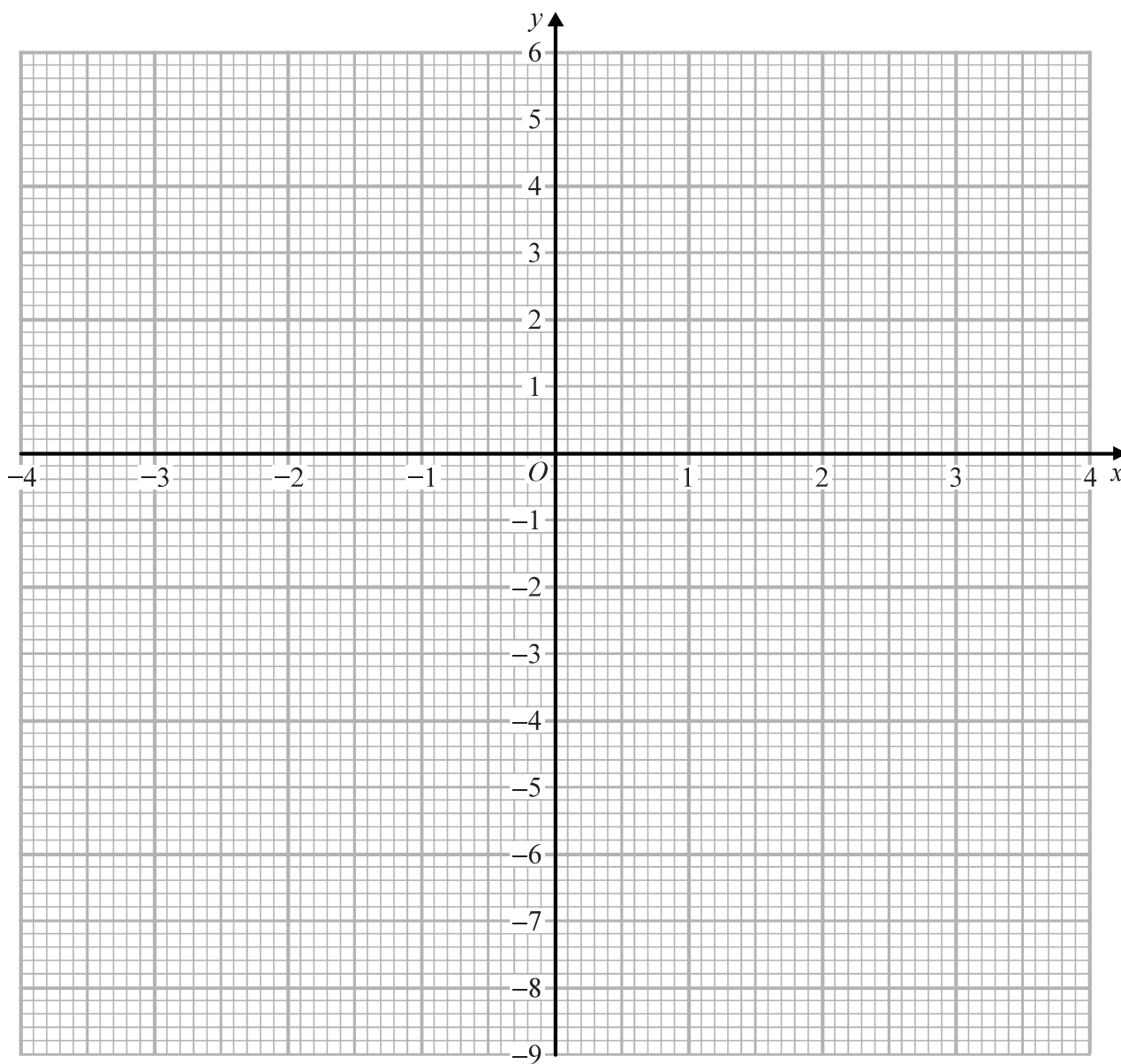
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Question 7 continued

Only use this grid if you need to redraw your curve.



(Total for Question 7 is 10 marks)



P 6 0 2 5 8 A 0 2 3 3 6

8 The functions f and g are defined as

$$f : x \mapsto 3x - 5$$

$$g : x \mapsto \frac{1}{2x - 3}$$

(a) State the value of x that must be excluded from any domain of g . (1)

(b) Find $gf(4)$. (2)

The function h is defined as

$$h : x \mapsto \frac{2x}{x - 3} \text{ where } x \neq 3$$

(c) Solve $fh(x) = 7$. (3)

(d) Express the inverse function h^{-1} in the form $h^{-1} : x \mapsto \dots$. (3)

(e) Solve the equation $g(x) + h(x) = 1$
Show your working clearly.

Give your solutions in the form $a \pm \sqrt{b}$ where a and b are integers. (5)

(a) The value of x that must be excluded is any value of x that makes the denominator $(2x-3)$ equal to 0 since dividing by 0 will give an imaginary number.

$$\begin{array}{l} 2x - 3 = 0 \\ +3 \quad \quad +3 \\ \hline 2x = 3 \\ \div 2 \quad \quad \div 2 \\ \hline x = \frac{3}{2} \end{array}$$

(b) To find $gf(4)$, you first need to find $gf(x)$ and then substitute 4.

$$\begin{aligned} gf(x) &= \frac{1}{2(3x-5)-3} \\ &= \frac{1}{6x-10-3} \\ &= \frac{1}{6x-13} \end{aligned}$$

$$\left[\text{Solutions of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$



Question 8 continued

Now substitute 4.

$$\begin{aligned}
 gf(4) &= \frac{1}{6(4) - 13} \\
 &= \frac{1}{24 - 13} \\
 &= \frac{1}{11}
 \end{aligned}$$

(c) First find $fh(x)$.

$$\begin{aligned}
 fh(x) &= 3\left(\frac{2x}{x-3}\right) - 5 \\
 &= \frac{6x}{x-3} - 5
 \end{aligned}$$

Now make $h(x)$ equal to 7 and solve for x .

$$\begin{aligned}
 \frac{6x}{x-3} - 5 &= 7 \\
 6x - 5(x-3) &= 7(x-3) \\
 6x - 5x + 15 &= 7x - 21 \\
 x + 15 &= 7x - 21 \\
 x + 36 &= 7x \\
 36 &= 6x \\
 6 &= x
 \end{aligned}$$

(d) To find $h^{-1}(x)$, you need to substitute $h(x)$ for y and then make x the subject of the equation.

$$\begin{aligned}
 y &= \frac{2x}{x-3} \\
 y(x-3) &= 2x \\
 yx - 3y &= 2x \\
 yx &= 2x + 3y \\
 yx - 2x &= 3y \\
 x(y-2) &= 3y \\
 x &= \frac{3y}{y-2}
 \end{aligned}$$



Question 8 continued

Now swap y for x and write as $h^{-1}:x$.

$$h^{-1}:x = \frac{3x}{x-2}$$

(e)

$$g(x) + h(x) = 1$$

$$\frac{1}{2x-3} + \frac{\frac{x-3}{x-3} \cdot \frac{2x}{x-3}}{\frac{x-3}{x-3} \cdot \frac{2x-3}{2x-3}} = 1$$

$$\frac{1(x-3) + 2x(2x-3)}{(2x-3)(x-3)} = 1$$

Expand

$$\frac{x-3 + 4x^2 - 6x}{2x^2 - 6x - 3x + 9} = 1$$

Simplify

$$\frac{4x^2 - 5x - 3}{2x^2 - 9x + 9} = 1$$

 $\times 2x^2 - 9x + 9$

$$4x^2 - 5x - 3 = 2x^2 - 9x + 9$$

 $-2x^2$

$$2x^2 - 5x - 3 = -9x + 9$$

 $+9x$

$$2x^2 + 4x - 3 = 9$$

 -9

$$2x^2 + 4x - 12 = 0$$

 $\div 2$

$$x^2 + 2x - 6 = 0$$

 $\times 2x^2 - 9x + 9$ $-2x^2$ $+9x$ -9 $\div 2$

Now use the quadratic formula to find two solutions for x .

$$\left[\text{Solutions of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

where $a = 1$ $b = 2$ $c = -6$

$$x_1 = \frac{-(2) + \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$$

$$= -1 + \sqrt{7}$$

$$x_2 = \frac{-(2) - \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$$

$$= -1 - \sqrt{7}$$



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Question 8 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 8 is 14 marks)



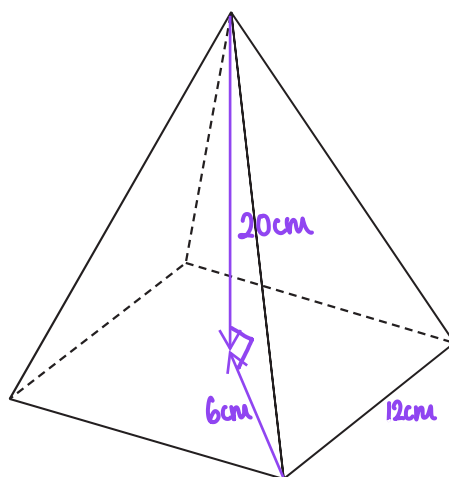


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Figure 2

Figure 2 shows a solid right square-based pyramid with height 20 cm. The volume of the pyramid is 960 cm^3 .

The pyramid is standing with its square base on a horizontal table.

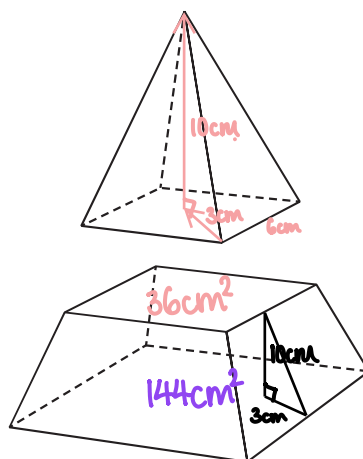


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Figure 3

The pyramid is divided into a smaller right square-based pyramid of height 10 cm and a frustum of the pyramid, as shown in Figure 3, by a horizontal cut.

- (a) Calculate the volume, in cm^3 , of the smaller pyramid. (3)
- (b) Calculate the total surface area, in cm^2 to 3 significant figures, of the frustum of the pyramid. (5)

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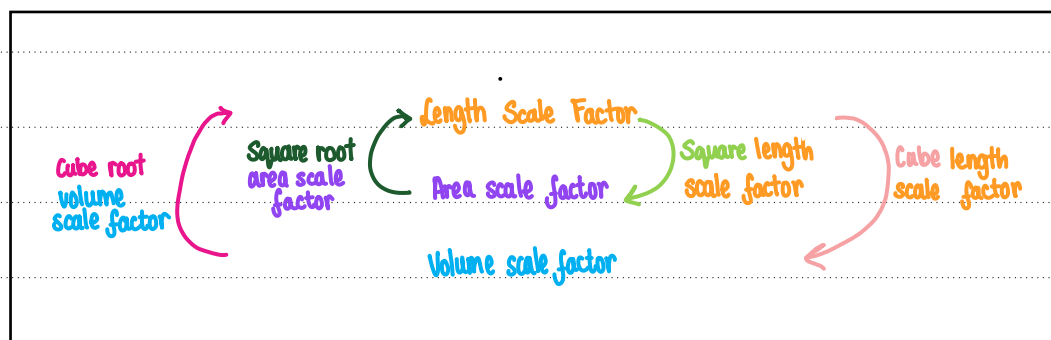
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Question 9 continued

(a) This is a scale factor questions.



Big pyramid : Small pyramid

Height 20cm : 10cm

$\times \frac{1}{2}$

Volume 960cm³ : 120cm³

$\times (\frac{1}{2})^3$

(b) To find the total surface area of the frustrum, you need to find the area of each square and trapezium.

To find the area of the bottom square, you need to work backwards from the equation for volume.

$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$960 = \frac{1}{3} \times \text{base area} \times 20$$

$$960 = \text{base area} \times \frac{20}{3}$$

$$144\text{cm}^2 = \text{base area}$$

You can find the area of the top square in the same way.

$$120 = \frac{1}{3} \times \text{base area} \times 10$$

$$120 = \text{base area} \times \frac{10}{3}$$

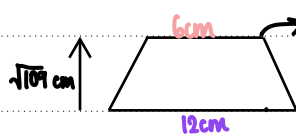
$$36\text{cm}^2 = \text{base area}$$

$$\left(\text{Volume of pyramid} = \frac{1}{3} \times \text{base area} \times \text{height} \right)$$



Question 9 continued

You can now find the area of the trapeziums using the base areas and the heights of the pyramids.



You can find this length with the area of the small square.

$$\text{Area of square} = \text{length}^2$$

$$36 = \text{length}^2$$

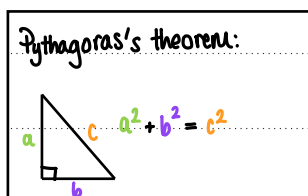
$$6 \text{ cm} = \text{length}$$

For the length of the longer side, work backwards from the area of the bigger square.

$$144 = \text{length}^2$$

$$12 \text{ cm} = \text{length}$$

You can find the height of the trapezium using Pythagoras's theorem.



$$3^2 + 10^2 = c^2$$

$$\sqrt{3^2 + 10^2} = c$$

$$\sqrt{109} \text{ cm} = c$$

$$\text{Area of trapezium} = \frac{1}{2} \times (a + b) \times \text{height}$$

$$\frac{1}{2} \times (12 + 6) \times \sqrt{109} = 9\sqrt{109} \text{ cm}^2$$

$$= 93.96... \text{ cm}^2$$

$$\text{Surface area} = 4(93.9...) + 144 + 36$$

$$= 555.85...$$

$$\approx 556 \text{ cm}^2$$



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Question 9 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 9 is 8 marks)

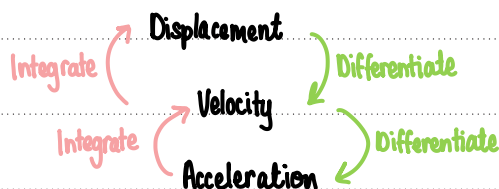


10 A particle, P , is moving along a straight line through the fixed point O .

The displacement, s metres, of P from O at time t seconds is given by

$$s = t^3 - 7t^2 - 5t + 121 \quad t \geq 0$$

Find the displacement, in metres, of P from O when P is instantaneously at rest.



Here you have to differentiate once for velocity and then find t for when $v=0$.

The general rule for differentiation is "bring down the power and subtract 1".

$$s = t^3 - 7t^2 - 5t + 121$$

$$v = (3)t^{3-1} - (2)7t^{2-1} - (1)5t^{1-1}$$

$$v = 3t^2 - 14t - 5$$

$$0 = 3t^2 - 14t - 5$$

You now factorise for t .

$$0 = (t - 5)(3t + 1)$$

$$0 = (t - 5)(3t + 1)$$

$$t - 5 = 0$$

$$3t + 1 = 0$$

$$t = 5$$

$$3t = -1$$

$$t = -\frac{1}{3}$$

Since $t > 0$, t can only be 5.

Substitute $t = 5$ into s .

$$s = (5)^3 - 7(5)^2 - 5(5) + 121$$

$$s = 46 \text{ m}$$



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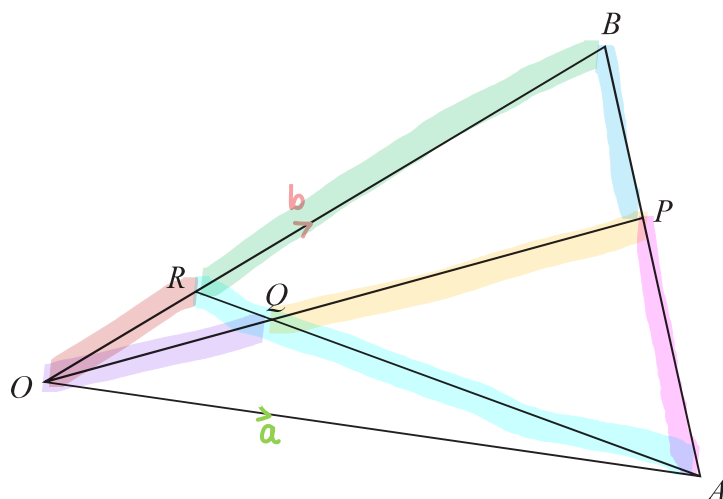


Diagram NOT
accurately drawn

Figure 4

Figure 4 shows triangle OAB in which $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

P is the point on AB such that $AP : PB = 2 : 1$

Q is the point on OP such that $OQ : QP = 1 : 3$

R is the point on OB such that RQA is a straight line.

Calculate, in its simplest form, the ratio $OR : RB$

First find the missing vectors on the diagram.

$$\vec{AB} = \vec{AO} + \vec{OB}$$

$$\vec{AB} = -\mathbf{a} + \mathbf{b}$$

$$\vec{AP} = \frac{2}{3}\vec{AB} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$$

$$\vec{PB} = \frac{1}{3}\vec{AB} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = \mathbf{a} + \frac{2}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$$

$$\vec{OP} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$$

$$\vec{OQ} = \frac{1}{4}\vec{OP} = \frac{1}{4}\left(\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}\right) = \frac{1}{12}\mathbf{a} + \frac{1}{6}\mathbf{b}$$

$$\vec{PQ} = \frac{3}{4}\vec{OP} = \frac{3}{4}\left(\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}\right) = \frac{1}{4}\mathbf{a} + \frac{1}{2}\mathbf{b}$$

$$\vec{AQ} = \vec{AP} + \vec{PQ}$$

$$\begin{aligned}\vec{AQ} &= \frac{2}{3}(\mathbf{b} - \mathbf{a}) - \left(\frac{1}{4}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \\ &= -\frac{11}{12}\mathbf{a} + \frac{1}{6}\mathbf{b}\end{aligned}$$



Question 11 continued

$$\vec{AR} = k(\vec{AB})$$

$$\vec{AR} = k\left(\frac{1}{6}b - \frac{11}{12}a\right)$$

$$\vec{OR} = \vec{OA} + \vec{AR}$$

$$\vec{OR} = a + k\left(\frac{1}{6}b - \frac{11}{12}a\right)$$

$$\vec{OR} = a + \frac{1}{6}kb - \frac{11}{12}ka$$

Since $\vec{OR}$ is parallel to $\vec{OB}$, a should equal 0.

$$a - \frac{11}{12}ka = 0a$$

$$1 - \frac{11}{12}k = 0$$

$$-\frac{11}{12}k = -1$$

$$k = \frac{12}{11}$$

$$\therefore \vec{OR} = \frac{1}{6}kb$$

$$\vec{OR} = \frac{1}{6}\left(\frac{12}{11}\right)b$$

$$\vec{OR} = \frac{2}{11}b$$

$$\vec{AR} = \frac{12}{11}\left(\frac{1}{6}b - \frac{11}{12}a\right)$$

$$\vec{AR} = \frac{2}{11}b - a$$

$$\vec{RB} = -\vec{AR} + \vec{AB}$$

$$\vec{RB} = -\left(\frac{2}{11}b - a\right) + -a + b$$

$$\vec{RB} = \frac{9}{11}b$$

$$\vec{OR} : \vec{RB}$$

$$\frac{2}{11} : \frac{9}{11}$$

$$2 : 9$$



Question 11 continued

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(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

